

Bounds on Heavy-to-Heavy Weak Decay Form Factors

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We provide upper and lower bounds on the semileptonic weak decay form factors for $B \rightarrow D^{(*)}$ and $\Lambda_b \rightarrow \Lambda_c$ decays by utilizing inclusive heavy quark effective theory sum rules. These bounds are calculated to second order in Λ_{QCD}/m_Q and first order in α_s . The $O(\alpha_s^2\beta_0)$ corrections to the bounds at zero recoil are also presented.

Form factors play an important role in both experimental measurements and theoretical calculations. In particular, they are often used to provide theoretical input for extraction of CKM matrix elements such as $|V_{cb}|$ and $|V_{ub}|$. However, the form factors used are not calculated from first principles but taken from models that have some additional assumptions. Therefore, it would be desirable if one could put constraints on the form factors that are free from any model dependence. Such bounds using heavy quark effective theory (HQET) inclusive sum rules had been derived.^{1,2} They have been further improved and applied to all form factors in $B \rightarrow D^{(*)}$ and $\Lambda_b \rightarrow \Lambda_c$ decays to first order in both $O(1/m_Q)$ and $O(\alpha_s)$ to the full spectrum and to $O(\alpha_s^2\beta_0)$ at zero recoil.^{3,4} They can provide a test for the models used in the literatures.

Using the optical theorem for the inclusive decay rate, one finds [†] taking the initial state to be a B meson as an example:

$$\begin{aligned} & \frac{1}{2\pi i} \int_C d\epsilon \theta(\Delta - \epsilon) T(\epsilon) \left(1 - \frac{\epsilon}{E_1 - E_H} \right) \\ & \leq \frac{|\langle H(v') | a \cdot J | B(v) \rangle|^2}{4M_B E_H} \leq \frac{1}{2\pi i} \int_C d\epsilon \theta(\Delta - \epsilon) T(\epsilon) \left(1 - \frac{\epsilon}{E_{max} - E_H} \right). \end{aligned} \quad (1)$$

One can readily obtain the corresponding formula for baryons by averaging over spins where appropriate. The middle part of Eq. (1) contains a hadronic matrix element that involves long distance physics. The goal is to find the proper 4-vector a_μ and weak decay current J^μ to project out the form factor combination that is of interest. On both sides of Eq. (1), one performs calculations in the partonic picture. The moments of $T(\epsilon)$ multiplied by the weight function $\theta(\Delta - \epsilon)$ can be computed perturbatively in QCD when the integration contour C is far from the cuts of physical processes. One also performs an operator product expansion

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[†]We follow the notation used by Chiang ⁴ where one can find a detailed derivation.

(OPE) in powers of the inverse heavy quark mass $1/m_b$ for $T(\epsilon)$. In general, if one performs the OPE for $T(\epsilon)$ to $O(1/m_b^n)$, then, due to consistency, both bounds are correct to $O(1/m^{n-1})$.⁴ The above-derived bounds have the features that: (i) the upper bound is model independent while the lower bound assumes that there is little contribution from multi-particle production, such as $B \rightarrow D \pi l \nu$, which is supported by experiments; (ii) the bounds can be applied to the whole kinematic regime in the case of heavy-to-heavy decays.

Schematically, our partonic calculations of the structure functions, T_i , can be expressed as:

$$T_i^{Full} \simeq T_i^1 + T_i^{1/m_Q} + T_i^{1/m_Q^2} + T_i^{1/m_Q^3} + \alpha_S [U_i + (\omega - 1)V_i] + \alpha_S^2 \beta_0 T_i^{\alpha_S^2 \beta_0}(\omega = 1), \quad (2)$$

where the first line of Eq. (2) is an expansion in powers of $1/m_Q$.⁵ We only expand the first order perturbation to terms linear in $\omega - 1$ because it is a good approximation within the allowed kinematic regime for heavy-to-heavy decays.^{2,3} The results for $T_i^{\alpha_S^2 \beta_0}(\omega = 1)$ evaluated at zero recoil are helpful in understanding the convergence of the perturbation series and are obtained using the method proposed by Smith and Voloshin.⁶ [‡]Corrections of order $(\frac{\Lambda_{\text{QCD}}}{m_Q})^3$, α_S^2 , $\alpha_S \frac{\Lambda_{\text{QCD}}}{m_Q}$, and $\alpha_S (\omega - 1)^2$ are neglected. Table 1 lists the expansion parameters used in the calculations:

Table 1. Expansion Parameters.

HQET parameters	
Λ_{QCD}	$\sim 0.5 \text{ GeV}$
$\bar{\Lambda}/M_B$	~ 0.1 for mesons; ⁷ ~ 0.15 for baryons
λ_1	$-0.19 \pm 0.10 \text{ GeV}^2$ for mesons; ⁷ $-0.43 \pm 0.10 \text{ GeV}^2$ for baryons
λ_2	0.12 GeV^2 for mesons; 0 for baryons
ρ_1, T_1 and T_2	$\sim \Lambda_{\text{QCD}}^3$ for mesons and baryons
ρ_2, T_3 and T_4	$\sim \Lambda_{\text{QCD}}^3$ for mesons; 0 for baryons
Perturbative parameters	
m_b	4.8 GeV
m_c	1.4 GeV
$\alpha_S(2 \text{ GeV})$	~ 0.3
Δ	1 GeV

As an example, the bounds on h_{A_1} are of particular interest because the form factor $F(\omega)$ that appears in $B \rightarrow D^* l \nu$ decay approximates h_{A_1} when $\omega \rightarrow 1$. Information on this can help determine the CKM matrix element $|V_{cb}|$.⁸ Here one can choose $a^\mu = (0, 1, 0, 0)$ and an axial current A_μ in Eq. (1) to form the bounds on $f(\omega) \equiv (\omega + 1)^2 |h_{A_1}|^2 / 4\omega$. Full analyses of other form factors and comparison with models often used or quoted in the literature are given elsewhere.⁹

At zero recoil, we find that to first order in $1/m_b$ both the upper and lower bounds on $f(1)$ coincide at 1, agreeing with Luke's theorem.¹⁰ When $O(\alpha_S)$ corrections are included, the bounds become $0.916 \leq f(1) \leq 0.927$ using the parameters

[‡]To utilize the method, one has to use a finite gluon mass to regularize the IR divergence

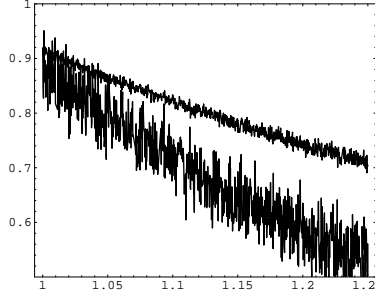


Fig. 1. Upper and lower bounds on $(\omega + 1)^2 |h_{A_1}(\omega)|^2 / (4\omega)$ in the regime $\omega \in (1, 1.25)$. The curves include $O(1/m_Q^2)$ and perturbative corrections. HQET parameters are varying: $\bar{\Lambda} \in (0.3, 0.5)$ GeV, $\lambda_1 \in (-0.1, -0.3)$ GeV², $\rho_{1,2} = \mathcal{T}_{1,2,3,4} \in (-0.125, 0.125)$ GeV³, and $\lambda_2 = 0.12$ GeV². Perturbative parameters are: $m_b = 4.8$ GeV, $m_c = 1.4$ GeV and $\Delta \in (1, 2)$ GeV.

given in Table 1. When uncertainties in the HQET parameters are taken into account, *i.e.* by varying the parameters $\bar{\Lambda} \in (0.3, 0.5)$ GeV, $\lambda_1 \in (-0.1, -0.3)$ GeV², $\rho_{1,2} = \mathcal{T}_{1,2,3,4} \in (-0.125, 0.125)$ GeV³ and keeping $\lambda_2 = 0.12$ GeV², the bounds get widened, as shown in FIG. 1. The bounds at zero recoil are roughly $0.84 \lesssim f(1) \lesssim 0.94$ where $O(\alpha_S^2 \beta_0)$ corrections at zero recoil are also included. Here the uncertainty is largely due to poor knowledge in the HQET parameters. The upper bounds solely depend upon λ_1 and $\bar{\Lambda}$, while the determination of the lower bounds are also affected by parameters showing up at $O(1/m_Q^3)$. Also, in order to have a better understanding of the bounds at the order being considered and at large ω , one should include $O(\alpha_S^2 \beta_0)$ corrections to the full spectrum.

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